TRINITY COLLEGE

YEAR 12
MATHEMATICS
APPLICATIONS

Test 2, 2023
Section One: Calculator Free
Sequences II and Networks

STUDENT'S NAME: Solutions [Francisco]

DATE: Thursday 4th May **TIME:** 25 minutes **MARKS:** 26
ASSESSMENT %: 10

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Formula Sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

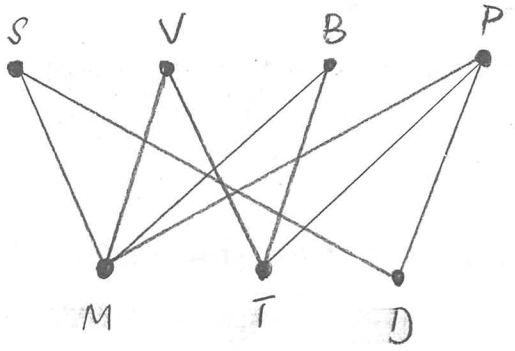
This page has been left intentionally blank

Question 1 **(6 marks)**

The table below shows the canteen purchases of three friends.

	Sausage Roll (S)	Vegemite Scroll (V)	BBQ Pizza (B)	Prime (P)
Matt (M)	✓	✓	✓	✓
Tom (T)		✓	✓	✓
Damon (D)	✓			✓

(a) Represent the information above as graph G that is clearly bipartite. (2 marks)



✓all vertices marked with dot labelled

✓all edges correct

(b) Is graph G a complete bipartite graph? Justify your response in the context of the question. (2 marks)

No. Tom and Damon will need to purchase all items for graph G to be a complete bipartite graph.

✓states no

✓suitable reasoning in context.

(c) Graph G can be drawn as a planar. Determine, with justification, the number of faces graph G has. (2 marks)

$7 + f - 9 = 2$

$f - 2 = 2$

$f = 4$

$\therefore$ 4 faces.

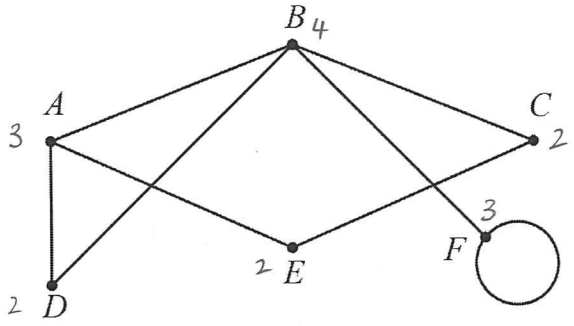
✓substitutes $v = 7$ and $e = 9$ into the equation

✓ solves for 4 faces.

6

Question 2 **(5 marks)**

The statements in parts (a) to (e) of this question relate to graph H shown below. For each statement, state whether it is true or false and support your answer with a clear justification.



(a) Graph H is a simple graph. (1 mark)

False, it contains a loop at vertex F.

✓ states false and mentions the loop

(b) Graph H is planar. (1 mark)

True, graph H can be redrawn as planar.

✓ states true and mentions that it can be redrawn as planar.

(c) Graph H has a bridge. (1 mark)

True, vertex BF is a bridge

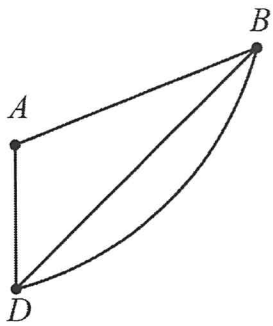
✓ states true and mentions edge BF.

(d) Graph H has 5 even vertices. (1 mark)

False, it has 4 even vertices.

✓ states false and mentions 4 even vertices

(e) Graph I, shown below, is a subgraph of graph H. (1 mark)



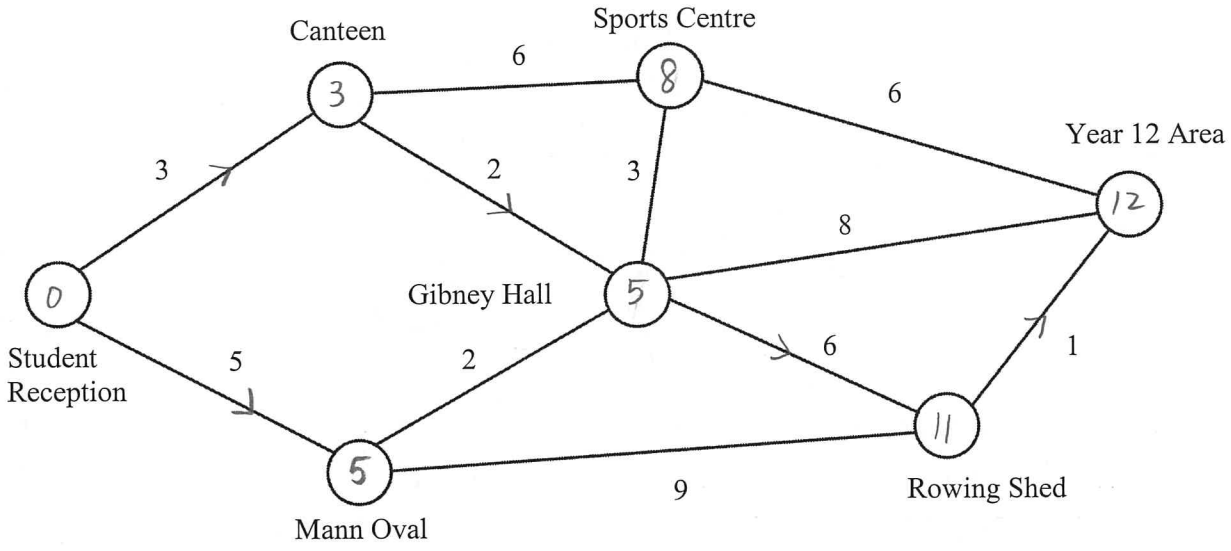
False, edge BD in graph H only has one edge.

✓ states false and mentions the extra edge in graph I that does not exist in graph H.

5

Question 3 **(9 marks)**

The network below shows the time taken, in minutes, to travel between locations in Trinity College.



Toby arrived late to school at 9:45 am on a Tuesday and after signing in at student reception, needed to put his belongings away into his locker in the Year 12 Area. Since he is already late to class, he will need to take the shortest path to get there.

(a) Determine the required shortest path and minimum time Toby should take from Student Reception to his locker. (3 marks)

Student Reception – Canteen – Gibney Hall – Rowing Shed – Y12 Area

12 minutes

✓ shows clear working on graph
✓ states correct shortest path
✓ states 12 minutes

(b) Determine how much time Toby would add onto his journey from Student Reception to his locker if he needed to make a stop at Mann Oval after visiting the Canteen. (2 marks)

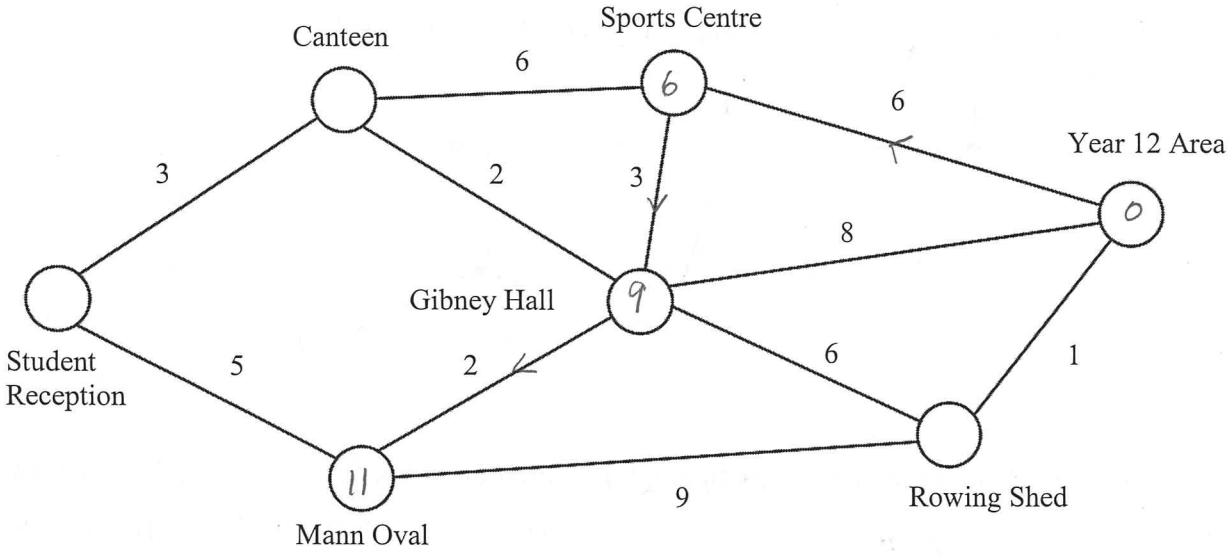
$3 + 2 + 2 + 2 + 6 + 1 = 16$ minutes.

∴ he adds 4 minutes

✓ calculates time taken for new path
✓ states 4 minutes.

5

After accessing his locker, Toby makes his way to his Period 2 Physical Education lesson at Mann Oval.



(c) Determine the earliest time that Toby can arrive to his Physical Education lesson if he needs to make a stop at the Sports Centre first. (2 marks)

Student Reception to locker = 9:57 am

9:57 + 11 min = 10:08 am

✓ calculates 11 minutes

✓ states 10:08am as earliest time.

(d) Toby realises that he left his hat at the Student Reception when signing in. He checks his diary to see the bell times (provided below). Will Toby be late to his Physical Education lesson on time if he is to retrieve his hat first? (2 marks)

Secondary School					
PERIOD	MON, WED & THURS	PERIOD	TUESDAY	PERIOD	FRIDAY
8.25am FIRST BELL					
Pastoral Care Group	8.30 – 8.40 (10 mins)	PCG Period	8.30 – 9.20 (50 mins)	Pastoral Care Group	8.30 – 8.40 (10 mins)
				Period 0	8.40 – 9.24 (44 mins)
Period 1	8.40 – 9.35 (55 mins)	Period 1	9.20 – 10.10 (50 mins)	Period 1	9.24 – 10.08 (44 mins)
Period 2	9.35 – 10.30 (55 mins)	Period 2	10.10 – 11.00 (50 mins)	Period 2	10.08 – 10.52 (44 mins)
Recess	10.30 – 11.00 (30 mins)	Recess	11.00 – 11.30 (30 mins)	Recess	10.52 – 11.22 (30 mins)

9:57 am at locker.

9:57 + 12 + 5 = 10.14 am

∴ Yes, he will be late by 4 minutes.

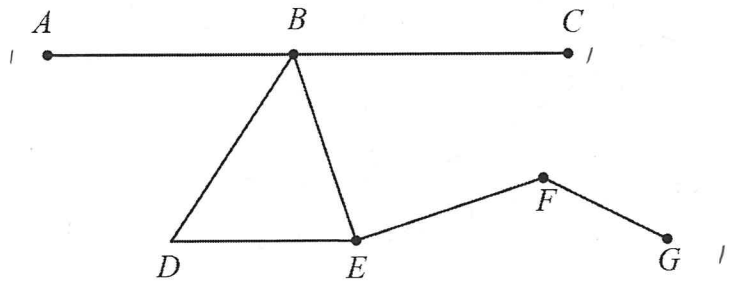
✓ calculates new arrival time 10.14 am

✓ states yes.

4

Question 4 **(6 marks)**

Graph J shown below represents the direct flights between 7 cities labelled $A - G$.



(b) List the vertices of the longest cycle in graph J starting at vertex B. (2 marks)

B D E B or B E D B

✓ lists a cycle
✓ starts at vertex B and ends

(c) Explain whether graph J is a Hamiltonian graph. (2 marks)

No it is not, as it does not contain a Hamiltonian cycle and it is not possible to include every vertex without repetition.

✓ states no
✓ mentions lack of Hamiltonian cycle or impossible to not repeat vertices.

(c) A new direct flight is to be added between two cities. With the addition of the new flight, graph J will be a semi-Hamiltonian graph. State all the possible pairs of direct flights that can be added. (2 marks)

A C , A G, A D , C G and C D ✓ states one correctly

✓ states all correctly.

END OF QUESTIONS

6

TRINITY COLLEGE

YEAR 12
MATHEMATICS
APPLICATIONS

Test 2, 2023
Section Two: Calculator Allowed
Sequences II and Networks

STUDENT'S NAME: _____

DATE: Thursday 4th May **TIME:** 25 minutes **MARKS:** 26
ASSESSMENT %: 5

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: 1 side of an A4 page notes, Classpad, Scientific Calculator and Formula Sheet

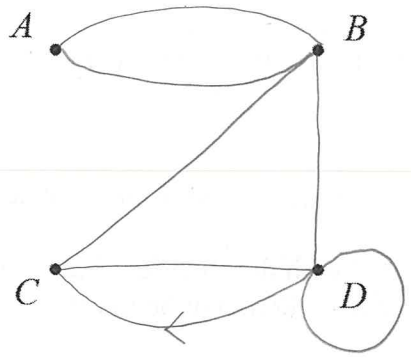
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

Question 5 **(3 marks)**

Consider the adjacency matrix below showing the direct paths of cities $A - B$.

	A	B	C	D
A	0	2	0	0
B	2	0	1	1
C	0	1	0	1
D	0	1	2	1

(a) Draw the connections represented in the matrix above as graph K. (2 marks)



✓ correct graph
✓ contains loop and direction at C to D.

(b) Is graph K a simple graph? Justify your answer. (1 mark)

No, it contains a loop and multiple edges

✓ states no and justifies by mentioning the loop or multiple edges

3

Question 6 **(8 marks)**

Matrix M, shown below, represents the road connection between suburbs in a particular city.

$$M = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 1 & 0 & 1 \\ C & 1 & 1 & 0 & 1 & 0 \\ D & 1 & 0 & 1 & 0 & 1 \\ E & 0 & 1 & 0 & 1 & 0 \end{array}$$

$$M^2 = \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 1 & 1 & 0 & 1 & 0 \\ B & 1 & 2 & 0 & 2 & 0 \\ C & 0 & 0 & 3 & 0 & 2 \\ D & 1 & 2 & 0 & 2 & 0 \\ E & 0 & 0 & 2 & 0 & 2 \end{array}$$

(a) If a network is to be drawn from matrix M, will the network be directed or undirected? (1 mark)

directed ✓ states directed

(b) State the value of the element in row 2, column 2 in matrix M and explain its significance. (2 marks)

$m_{22} = 0$. ✓ states 0

There are 0 walks of length 1 to go from B to B.

✓ states 0 walks/ways using one step / length 1 from B to B.

(c) Determine the number of ways to travel from suburb C to suburb D using a walk of length 1. (1 mark)

One way. ✓ states 1

(d) Determine the number of ways to travel between suburb A and suburb D using a walk of length 2 starting at either vertex. (1 mark)

Two ways. ✓ states 2

The mayor of the city decided that each suburb needs to be connected to every other suburb.

(e) State the terminology associated with the situation that the mayor has proposed and hence, decide how many road connections there will be in total. (3 marks)

A complete graph. ✓ states complete graph

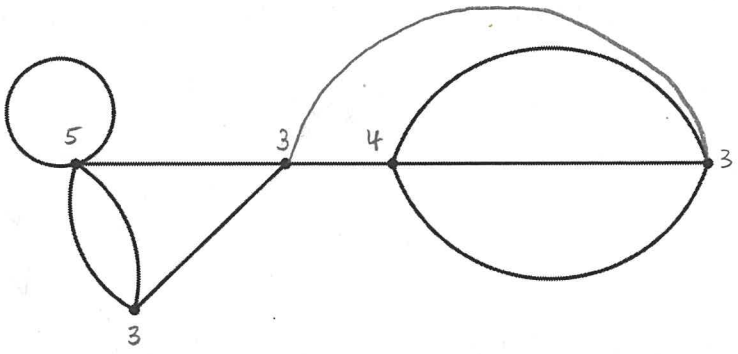
$\frac{5(5-1)}{2} = \frac{5 \times 4}{2} = 10$ ✓ substitutes $n=5$ into $\frac{n(n-1)}{2}$

∴ 10 road connections. ✓ solves for 10 connections

8

Question 7 **(6 marks)**

(a) Consider the graph below.



(i) State, with justification, whether the graph above is Eulerian, semi-Eulerian or neither. (2 marks)

Neither. It does not contain an Eulerian trail and it is not possible to travel every edge without repeating edges.

✓ states neither
✓ mentions that it does not contain an Eulerian trail or it has 4 odd degrees

(ii) Draw one edge on the graph above so that it becomes semi-Eulerian and does not contain a bridge. (1 mark)

✓ draws correct edge

(b) Consider a connected undirected graph, K, which has the following features:

- It is semi-Eulerian.
- It has 4 vertices.
- It has only 2 pairs of vertices each with identical degrees.
- The vertices which are not identical has a difference of 3, where the degree of the odd vertices is larger than the even vertices.
- The sum of the degrees is $4n + 26$.

With mathematical justification, determine the degrees of each vertex in graph K.
Let the degree of the even vertices be $2n$. (3 marks)

$2n + 2n + 2n+3 + 2n+3 = 4n + 26$

✓ creates an equation for sum of the degrees.

$8n + 6 = 4n + 26$

✓ solves equation

$4n = 20$

$n = 5$

✓ states degrees of each vertex.

∴ The vertices have a degree of 10, 10, 13 and 13.

6

Question 8 **(9 marks)**

Luca's car has an oil leak. At the start of the week, he notices that the oil level has decreased by 35%, so he adds 550 mL of oil to combat the situation. His car has a maximum oil capacity of 5 L. Luca produced the following sequence to represent the amount of oil his car has at the start of each week, in mL:

$$T_{n+1} = aT_n + b, \qquad T_0 = 4200$$

(a) State the values of a and b. (2 marks)

$a = 0.65$ and $b = 550$

✓ $a = 0.65$
✓ $b = 550$

(b) When does the oil level first reach half of its maximum capacity? (1 mark)

$T_2 = 2682$
$T_3 = 2293.3$

∴ At the start of week 3.

✓ states start of week 3.

(c) Describe, with mathematical justification, what is happening to the oil level in the long term. (2 marks)

It is decreasing and reaching a steady state of 1571.4 L

✓ states decreasing
✓ states reaches steady state of 1571.4 L

(d) Calculate the amount of oil that needs to be added at the start of each week for Luca to always maintain 75% of the maximum oil capacity. (2 marks)

$0.75 \times 5000 = 3750$

✓ calculates 3750 mL

$0.35 \times 3750 = 1312.5$ mL

✓ calculates 1312.5 mL

(e) Given that Luca decided to instead, add 1.8 L of oil at the start of every week, can the recursive sequence Luca produced continue indefinitely? Justify your response mathematically. (2 marks)

No, if he adds 1.8 L every week, the oil level in his car will increase and reach a steady state of 5142.9 L, exceeding the maximum capacity.

✓ states no
✓ explains steady state exceeds maximum oil capacity.

END OF QUESTIONS

9